

The decline in US public firms^{*}

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PRELIMINARY

ABSTRACT

Since its peak in 1996, the number of publicly listed US firms has declined by approximately 50%. In addition, US publicly listed firms are now on average larger and older than they were two decades ago. We develop a model to evaluate which of two mechanisms — an increase in the cost of being public or a reduction in the cost of capital for private firms — can explain the decline in US public listings and change in public firm distribution. The model features both private and public firms where firms optimally choose the timing of their IPO and exit. We calibrate the model to match the data prior to 1996 and then illustrate how changes in the private firm discount rate and public firm operating cost affect the cross-sectional distributions of public and private firms.

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1 Introduction

Recent literature has documented a substantial decline in the number of US public firms over the last two decades. Since its peak of more than 7,500 in 1996, the number of publicly listed companies is now less than 3,700, the same as it was forty years ago. In contrast, the total number of US firms has steadily increased. Moreover, with a lower frequency of initial public offerings, fewer young and small firms are going public. Taken together, these empirical facts point to deficiencies in US public equity markets, such as increasingly strict regulation. At the same time, the decline in public firms could follow from improvements in US private capital markets. Alternatively, a contributing factor could be a compositional change, where there is a drop in the types of firms for which going public is beneficial. While a number of potential candidates have been suggested, yet no consensus exists on which of the ultimate causes are responsible for the decline.

The goal of this paper is to identify the likely causes of the decline in public firms and evaluate how much the increased cost of being a public company reflects a problem with public equity markets. We focus on two commonly proposed mechanisms. On one side, the regulatory changes of the early 2000s resulted in higher compliance costs on public firms. One prominent example is the Sarbanes Oxley Act, which made disclosure requirements stricter and increased the administrative costs of preparing accounting statements (Leuz (2007), Zhang (2007), Engel et al. (2007), Iliev (2010)). However, increased regulatory costs can not account for the low number of public listings by itself as the drop starts in 1997 before the regulatory changes begin as noted in Kahle and Stulz (2017). On an another side, the decline in public firms can stem from increased inflows of private equity financing as well as improved conditions in private markets that allow for more liquidity in the equity holding of investors.

To uncover the causes of the decline in public firms, we develop a tractable real option model that captures the tradeoffs to a company being publicly versus privately owned. All companies are initially born as private and can subsequently decide to access public capital markets by paying a one-time fixed IPO cost. Firms generate stochastic cash flows and

incur a fixed ongoing cost of operation. For a sufficiently high level of cash flows, private firms optimally choose to go public – when the public firm valuation, net of the entry cost, exceeds the private firm valuation. The presence of fixed costs generates an optimal abandonment threshold both for private and public firms. Using these endogenous thresholds for an IPO and exit, we derive valuations of firms, as well as characterize their stationary distributions. Importantly, a regime shift in model parameters can be mapped into the proposed mechanisms responsible for the decline.

In the model, public firms incur a relatively higher ongoing cost of operation as they face such additional costs as publishing quarterly financial reports and communicating with shareholders (Bushee and Miller (2012)). We capture a stricter regulation post early 2000s in public markets with an increase in the public firms operational cost. Upon an increase, private firms optimally choose to go public later, resulting in a higher cash flow threshold for an IPO and, consequently, in a lower IPO rate. At the same time, a higher cost of being publicly owned causes public firms to exit sooner. The model also assumes that investors are risk-neutral and discount firms cash flows at constant rate. Improved conditions in private markets can be captured with a decreased in the cost of capital for private firms. Similarly to an increased operating cost for public firms, a lower private firm discount rate delivers a higher IPO threshold, as private firms valuations are relatively higher. However, upon this shift the exit threshold remains unchanged.

We next assess the tradeoffs to a company being publicly versus privately owned quantitatively. We therefore calibrate the model parameters to best match patterns in the US public markets. To this end, we organize a focused set of empirical facts that shape the story of the decline in public firms. First, we construct the propensity to go public, defined as the ratio of the number of public firms to the number of private firms. We document that the propensity to go public declines roughly as much as the number of public firms, indicating that the trend in public firms is not a byproduct of a more widespread dip in the number of private firms. Second, we examine public firms turnover. Though we observe heightened exit rates around 2000, the decline in the number of public firms is equally driven by a substan-

tial decline in the IPO rate. Third, we investigate the shifts in the public firm distribution. Since 1996, the typical public firm is twice as large. The right tail of the distribution has increased, though not as much as the typical firm, while the left tail has decreased. Finally, we demonstrate that not only does the typical public firm becomes larger, but so does the typical firm that goes public, both in level and relative to existing public firms.

We calibrate the parameters of the model for the period 1980–1996, when the number of US publicly listed firms was increasing. Then we use the calibrated model to evaluate the effects of three parameter changes. The first is a reduction in the private firm discount rate, reflecting an improvement in private capital markets. The second is an increase in the fixed costs of being publicly listed, reflecting an increase in the regulatory and disclosure burdens facing public firms. The final change we consider is an increase in the sunk cost of becoming a public firm. These costs reflect both the direct and indirect costs of a firm’s IPO as well as the costs of adjusting to being a public firm.

Our paper contributes to several strands of literature. First and foremost, our research project complements recent studies that document the decline in the number of US public firms, the so-called “US listing gap,” and investigate possible explanations for it. Doidge et al. (2017) document that these empirical patterns are novel to the US: the number of public listings has not declined in other countries, either developed or developing.

A growing body of work has identified some key features associate with this decline. As shown in Kahle and Stulz (2017), in the recent years the set of public firms has become larger, older, and more reliant on R&D relative to capital investment. Doidge et al. (2017) find that the low number of listings comes approximately equally from a low number of new lists and a high number of delists. Accordingly, Gao et al. (2013) document that the IPO rate is particularly low among small, young firms that are disappearing from public markets. Collectively, these empirical papers point to the possibility that something is amiss in US public markets. Our paper sheds light on whether the decline in public listings is a symptom of a broader issue with public markets or is a result of increased private equity financing and improved conditions in private markets. In this sense, our work is a key step

towards informing future economic policies and regulations targeted at improving the quality of public capital markets.

More broadly, our paper relates to the literature studying the costs and benefits that enter firms decision to go public. Key first order factor is that public ownership of equity allows firms to grow and obtain necessary financing at a scale that is not possible for a privately owned company. Yet, they must pay a large fixed cost to do so (Lowry et al. (2017)) and incur additional ongoing costs stemming from regulation and disclosure requirements. In our model, these aspects are directly embedded in the firms choice to go public.

Companies that enter public capital markets face increased visibility, along with its advantages and disadvantages. Because public investors tend to be more diversified, they might be willing to pay a higher price for public ownership of equity compared to private investors, allowing firms to sell their shares at a higher price (Chemmanur and Fulghieri (1999)). However, increased visibility can attract additional competition in the product market (Maksimovic and Pichler (2001)).

There are other considerations related to acquisitions and control. Zingales (1995) argues that going public makes it easier to find a potential buyer to acquire the firm. Others argue the reverse, that firms conduct an IPO in order to more easily acquire other firms (Brau and Fawcett (2006), Celikyurt et al. (2010)). On a different note, firms may choose to go public in order to divert ownership away from venture capitalists and re-establish control of the firm (Black and Gilson (1998)). Though these considerations might affect the firms decision to IPO, there is no evidence of structural shifts in these incentives.

Lastly but not least, our paper fits into the literature that models the firm's IPO decision, including Chemmanur and Fulghieri (1999), Bayar and Chemmanur (2011), Begenau and Palazzo (2017). In contrast to these studies, we develop a real option model of a firm's decision to go public that allows us to characterize the cross-sectional distribution of public and private firms.

The rest of the paper is organized as follows. Section 2 discusses the data and presents empirical facts. Section 3 describes the model. Section 4 performs a calibration and shows

the effects of changes in the private firms' cost of capital and costs of being public. Finally, Section 5 concludes.

2 Data and Empirical Facts

2.1 Data and Empirical Measurements

To measure the number of publicly listed firms in the U.S., we use the Center for Research in Security Prices (CRSP) Monthly Stock File and follow the definition of Doidge et al. (2017). Specifically, we include all U.S. domiciled common stocks listed on the NYSE, AMEX, and NASDAQ stock exchanges, except the investment funds and trusts. We keep only December observations, assuming that the presence in the last month of a given year indicates the firm's existence in that year. Our benchmark sample covers the period from 1980 until 2016.

To understand how public firms have changed over the sample period, we merge our CRSP dataset with the Standard & Poor's Compustat Annual. We keep not only matched, but also unmatched Compustat observations that satisfy the above criteria. The dataset provides firm's total revenues (*sale*), total book assets (*at*), debt in current liabilities (*dlcc*), long-term debt (*dlt*), and number of employees (*emp*)¹. For some firms, Compustat reports financial data few years prior to the initial public offering. Backfilled data can bias upward the number of publicly listed firms. To tackle this issue, we use the offer dates for firms going public collected by Jay R. Ritter and exclude observations prior to the firm's first trading day. Moreover, we use the Ritter founding dates to construct firm age.

To characterize the distribution of U.S. private firms, we use business tax statistics prepared by the Internal Revenue Services (IRS). The IRS provides balance sheet, income statement, and other selected financial data for all active corporations filing Form 1120 from 1994 until 2013. Additionally, it presents data for corporations filing Form 1120S (i.e. S-corporations) but for the shorter period starting from 1998. The data are available at the aggregate level and for the subsets of firms classified by the size of business receipts. Our

¹We replace *sale*, *at*, *dlcc*, *dlt*, and *emp* with a missing value when they are less than or equal to zero.

goal is to define a set of private firms that are very likely to consider whether to go public or stay private. As such, we exclude from our analysis firms incorporated as S-corporations. Further, we restrict our attention to sufficiently large private firms, since these are the firms with the resources necessary to go public and maintain a public listing. In particular, we focus on firms with total revenues above \$10m and \$50m.

We face another challenge with characterizing the distribution of private firms. The IRS classifies corporations into size groups based on nominal cutoff values for total assets and business receipts, making it difficult to compare private firms' characteristics across years. For instance, we can observe the growth in the number firms with the nominal revenues above \$50m, even if the number of large firms with the revenues above the corresponding real cutoff values remains constant. Such growth would be purely driven by inflation, rather than by growth of large businesses.

To address this concern, we complement our analysis with firm counts from the U.S. Census Bureau based on the number of employees. Specifically, we focus on firms with more than 500 employees. Importantly, these counts are not subject to inflation issues. However, using Census data limits our analysis of the firms' financial characteristics.

In addition to firms' financial data, we also use data on CPI inflation from the Bureau of Labor Statistics. The price level is normalized to 1 in December of 2005. All nominal quantities are deflated by the CPI to obtain real measures.

2.2 Empirical Evidence

2.2.1 Propensity to Go Public

In this section, we revisit the evolution of U.S. public listings over the past few decades. Figure 1 shows that the number of public firms has increased rather steadily from 1980 until 1996 and then decreased almost twofold since 1996. In their paper, Doidge et al. (2017) document that this dramatic decline is unique to the United States and has not arisen in the rest of the world, constituting the so-called U.S. listing gap.

We further investigate whether this decline is specific to publicly listed firms or applies

to private ones as well. To this end, we construct the firm propensity to go public over time, defined as the ratio of the number of public firms to the number of private firms. Note that if the decrease in the number of public firms coincides with a corresponding decrease in the number of private firms, this ratio would remain constant. Instead, Panel B of Figure 1 displays substantial growth in the number of private firms with more than 500 employees over the period from 1994 until 2013, ruling out the possibility that the decrease in the number of public firms is the product of a widespread downwards trend in the number of firms. Accordingly, due to the number of public and private firms moving in opposite directions, we observe a steady decline in the firm's propensity to go public (see Panel C of Figure 1). Importantly, this trend is quantitatively and qualitatively robust to using firms with more than 500 employees versus firms with total revenues above \$50m. In the earlier case, the firm's propensity to go public has decreased by 63% over the period from 1998 until 2013, while in the latter case – by 57%.

The decline in the number of public listing in the U.S. may have resulted either from a decline in the IPO rate or from an increase in the exit rate. Figure 5 plots the number of IPOs and IPO rate over time, where the latter is calculated as the ratio of the number of IPOs to the number of private firms with more than 500 employees. The IPO rate declines from 8% in 1996 to 1% in 2013, indicating that the decline in the number of public firms can be partially explained by the decline in the IPO rate.

Figure 6 (Panel A) depicts the number of public firms that have delisted from a stock exchange either voluntarily or involuntarily. As can be seen from the figure, the number of delists has been fluctuating between 200 and 500 firms per year in the pre-1996 period. However, it has increased dramatically thereafter, reaching its peak of 875 in 1998 and then reverting back to near the pre-1996 values. This large increase in firm exits in the late '90s and early '00s has contributed to the decline in the number of public listings. At the same time, historically low number of delists since 2004 cannot solely explain a further drop in the propensity to go public. We also plot the number of delists scaled by the total number of public firms, i.e. the exit rate (see Panel B of Figure 6). The dynamics of the exit rate

follow very closely the dynamics of the raw number of firm exits.

We further examine the delisted firms by decomposing the exit rate into reasons for delist. First, we examine delists for “negative” reasons (e.g. company liquidation or bankruptcy), defined as securities with delisting codes $4xx$ and $5xx$. Panel D demonstrates that the exit rate for “negative” delists has been fluctuating mostly between 2% and 5%, without exhibiting any secular trends. Second, we examine delists for mergers and acquisitions, defined as securities with delisting codes $2xx$ and $3xx$. Panel F demonstrates that the exit rate for mergers and acquisitions fluctuates between 2% and 6%, with sharp increase in the early '90s until 2000, dip in 2001, and end value around the 1996 rate.

2.2.2 Size Distribution of Public Firms

Over our sample period, there has been a significant shift in the distribution of publicly listed U.S. firms. We find that over the last few decades the typical public firm has become larger. As shown in Figure 2, the median revenue of public firms has declined threefold from \$412m in 1980 to \$134m in 1996, and has increased twofold from \$134m in 1996 to \$262m in 2016. We document a similar U-shape pattern when measuring firm size using total book assets or number of employees (see Appendix Figure A.1). A different pattern emerges when measuring firm size using market value of equity, market value of the firm, or age. Using these we observe the steady increase in the median firm’s size over the period from 1980 until 2016.

To further characterize how public firms have changed over time, we look at the behavior in the tail ends of the distribution. To this end, we construct two measures. First, we calculate the revenues in the bottom and top percentiles and plot these against median firm size, where we measure size using total real revenues. By analyzing changes relative to the typical firm, we avoid capturing lateral shifts in the entire distribution of public firms. Figure 3 demonstrates that public firms in the bottom percentiles have become smaller, especially when compared to a median public firm. At the same time, public firms in the top percentiles have become larger, with a smaller growth rate than the median public firm’s growth rate.

Second, we analyze how revenue and market value shares from firms in the top and bottom percentiles have changed over the sample period. As shown in Panels A and C of Figure 4, both revenue and market value shares from smallest public firms have declined from prior to the decline in public firms to the end of the sample. There were two contributing factors: the smallest firm has become smaller and the median public firm has become larger. Similarly, we observe declines in the revenues and market value shares from largest public firms, but they are of much smaller magnitude (see Panels B and D of Figure 4). This is despite the fact that the public firms in the top percentiles have become larger, meaning that the median firm experienced a slightly higher growth rate than the largest firms.

2.2.3 Firm Characteristics at IPO

Not only does the typical public firm becomes larger in the recent years, but so does the typical firm that goes public. As shown in Figure 7, firm size at IPO year increases almost threefold from prior to the decline in public firms to the end of the sample. This finding continues to hold if we measure the firm's size with total book assets, market equity and firm value, number of employees, and age (see Figure A.2).

A larger size threshold needed to IPO could be indicative of a lower net benefit of going public, but likely not if the underlying reason is a shift to the right in the firm size distribution. As such, we investigate by how much firm size at IPO year has changed relative to other public firms. Specifically, we identify the percentile of median firm revenue at IPO year within the distribution of public firms. As shown in Panel A of Figure 8, in the beginning of the sample firms conducting IPOs are larger than 30-40% of public firms, increasing to 40-60% of public firms by the end of the sample. We find similar patterns when measuring firm size using market value. This behavior suggests that the firms conducting IPOs are not only larger, but also larger relative to existing public firms.

3 Model

In this section, we describe the model setup and derive valuations for private and public firms. We then characterize the stationary distributions of public and private firms and provide comparative statics to illustrate the influence of various parameters of the model.

The economy is populated with a continuum of firms, consisting of two types: public and private. All firms are initially born as private and can subsequently choose to become public by paying a fixed cost. Firms generate cash flows

$$X_t - c, \tag{1}$$

where X_t evolves as a geometric Brownian motion,

$$\frac{dX_t}{X_t} = \mu dt + \sigma dW_t, \tag{2}$$

with $X_0 > 0$ and where c is a fixed cost of operation. Given this fixed operating cost, for some level of cash flows, firms optimally choose to exit. Additionally, firms are subject to a death shock that occurs as a Poisson arrival with intensity λ . We assume firms have zero recovery value at exit. Investors are risk-neutral, but apply a higher discount rate to private firms, reflecting the illiquidity of private, relative to public, equity. We allow public and private firms to differ in these five parameters $(\mu, \sigma, r, \lambda, c)$ and we use a subscript “1” on these parameters to denote private firms and “2” to denote public firms.

3.1 Private firms

Let $V(X)$ denote the value of a private firm with current cash flow level X . For a sufficiently low level of cash flow, denoted X_D , a private firm optimally chooses to shut down. For a sufficiently high level of cash flow, denoted \bar{X} , the private firm decides to go public by paying a one-time fixed cost of \mathcal{I}_{IPO} . Thus, the private firm’s value, $V(X)$, is defined on the interval (X_D, \bar{X}) . Following standard arguments (see, e.g., Dixit and Pindyck (1994)), a private firm’s valuation satisfies the ODE:

$$r_1 V = \mu_1 X V_X + \frac{\sigma_1^2}{2} X^2 V_{XX} + X - c_1 - \lambda_1 V. \tag{3}$$

This ODE has the general solution

$$V(X) = A_1 X^{\gamma_1} + A_2 X^{\gamma_2} + \frac{X}{r_1 + \lambda_1 - \mu_1} - \frac{c_1}{r_1 + \lambda_1}, \quad (4)$$

where $\gamma_1 > 1$ and $\gamma_2 < 0$ are the solutions to the fundamental quadratic for the ODE:

$$\gamma = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r + \lambda)}{\sigma^2}}. \quad (5)$$

Optimal exercise of the exit and IPO options give the following boundary conditions:

$$V(X_D) = 0 \quad (6)$$

$$V_X(X_D) = 0 \quad (7)$$

$$V(\bar{X}) = W(\bar{X}) - \mathcal{I}_{IPO} \quad (8)$$

$$V_X(\bar{X}) = W_X(\bar{X}), \quad (9)$$

where $W(X)$ denotes the value of a public firm, to be defined below. Equations (6) and (7) are the value-matching and smooth pasting conditions for the firm's optimally chosen cash flow threshold, X_D , at which it exits. Similarly, equations (8) and (9) are the value-matching and smooth-pasting conditions for a private firm's optimal choice of cash flow threshold \bar{X} at which to undertake and IPO and become public. These four equations determine the optimal abandonment threshold, X_D , optimal IPO threshold, \bar{X} , and coefficients A_1 and A_2 .

3.2 Public firms

Public firms have cash flows $X_t - c_2$, where X evolves according to

$$\frac{dX_t}{X_t} = \mu_2 dt + \sigma_2 dW_t. \quad (10)$$

Public firms are also subject to death shocks with intensity λ_2 . The value of a public firm, $W(X)$, satisfies the ODE:

$$r_2 W = \mu_2 X_t W_X + \frac{\sigma_2^2}{2} X^2 W_{XX} + X - c_2 - \lambda_2 W \quad (11)$$

This has a general solution

$$W(X) = B_1 X^{\xi_1} + B_2 X^{\xi_2} + \frac{X}{r_2 + \lambda_2 - \mu_2} - \frac{c_2}{r_2 + \lambda_2}. \quad (12)$$

where $\xi_1 > 1$ and $\xi_2 < 0$ are the solutions to the fundamental quadratic for the ODE:

$$\xi = \frac{1}{2} - \frac{\mu_2}{\sigma_2^2} \pm \sqrt{\left(\frac{\mu_2}{\sigma_2^2} - \frac{1}{2}\right)^2 + \frac{2(r_2 + \lambda_2)}{\sigma_2^2}}. \quad (13)$$

To ensure the valuation is bounded, we require the coefficient on the positive root be set to zero. Public firms optimally choose to exit for a sufficiently low level of cash flow, $X_{D,public}$. Note that this exit threshold is different from the exit threshold for a private firm. The value-matching and smooth pasting conditions for the public firm's optimal exit decision are

$$W(X_{D,public}) = 0 \quad (14)$$

$$W_X(X_{D,public}) = 0. \quad (15)$$

Solving, we get the optimal exit threshold for public firms,

$$X_{D,public} = \frac{\xi_2}{\xi_2 - 1} \frac{(r_2 + \lambda_2 - \mu_2)c_2}{r_2 + \lambda_2}, \quad (16)$$

and the value of a public firm as a function of its current cash flow level:

$$W(X) = \frac{X}{r_2 + \lambda_2 - \mu_2} - \frac{c_2}{r_2 + \lambda_2} + \left(\frac{c_2}{r_2 + \lambda_2} - \frac{X_{D,public}}{r_2 + \lambda_2 - \mu_2} \right) \left(\frac{X}{X_{D,public}} \right)^{\xi_2}. \quad (17)$$

This expression for the public valuation can be plugged back in to equations (8) and (9) to solve for the coefficients in the private firm's valuation.

Before proceeding to characterize the distributions of public and private firms, we present comparative statics for the optimal thresholds for public firm entry (IPO) and exit. Figure 9 shows the optimal IPO threshold, \bar{X} , as a function of the underlying parameters. For a higher growth rate of private firm cash flows, holding fixed the growth rate of public firm cash flows, a firm waits longer to go public, increasing \bar{X} . Conversely, when public firm cash flows have a higher growth rate, going public looks more attractive and firms optimally choose a lower \bar{X} . As private firm volatility increases, the option to delay the IPO becomes more valuable, resulting in a higher \bar{X} . For a higher private firm discount rate (r_1), probability of an exit shock (λ_1), or fixed operating cost (c_1), a firm will choose to go public sooner, resulting in a lower IPO threshold. Finally, for a higher fixed cost of IPO, a firm delays the decision to go public.

Figure 10 shows comparative statics for $X_{D,public}$, a public firm's choice of exit threshold. Intuitively, for a higher mean or volatility of cash flow growth, a public firm optimally chooses to delay exit, resulting in a lower $X_{D,public}$. A higher discount rate, exit shock probability, or fixed operating cost, result in a public firm choosing to exit sooner. We see that private firm parameters, as well as the fixed cost of going public, \mathcal{I}_{IPO} , do not affect a public firm's choice of exit. While the private firm parameters and cost of IPO will affect firms' decision to go public and therefore the size distribution of public firms, once public these parameters are irrelevant for the exit decision. In this sense, we can think about the costs of IPO as more broadly capturing sunk costs that are part of the transition to becoming a public firm, while c_2 captures ongoing costs of being public.

3.3 Distribution of private firms

There is an exogenous flow N_t of new private firms that enter. Private firms exit for one of three reasons: they reach the optimal exit threshold $X_{D,private}$, the IPO threshold \bar{X} , or are hit with an exogenous death shock. Define $x \equiv \log(X)$ and note that by applying Itô's Lemma, it can be shown that x evolves as an arithmetic Brownian motion given by

$$dx_t = \left(\mu - \frac{1}{2}\sigma^2 \right) dt + \sigma dW_t. \quad (18)$$

Let $\phi(x)$ denote the stationary distribution of log cash flows for private firms. Firms exogenously exit at rate λ . A new private firm can choose to enter by paying a cost $I_{private}$ and draws its initial log cash flow from a distribution denoted $g(x)$. We assume that the distribution of initial cash flows of private firms entrants is uniform: $X_{entry} \sim U[X_a, X_b]$, where $X_D < X_a < X_b < \bar{X}$. This implies that the log cash flow distribution of private firm entrants, $g(x)$, is exponentially distributed over the interval $[x_a, x_b]$:

$$g(x) = e^{x-\hat{x}}, \quad (19)$$

where $\hat{x} = \log(X_b - X_a)$. To solve for the stationary distribution of log cash flows, $\phi(x)$, we consider three separate cases: $x \in (x_D, x_a)$; $x \in (x_a, x_b)$; $x \in (x_b, \bar{x})$.

Case 1: $x \in (x_D, x_a)$

Over this interval, firms exit at rate λ , however there is no flow of new entrants. The Kolmogorov forward equation (KFE) characterizing the steady state distribution for this region satisfies

$$\frac{1}{2}\sigma^2\phi_{xx}(x) - \left(\mu - \frac{1}{2}\sigma^2\right)\phi_x(x) - \lambda\phi(x) = 0. \quad (20)$$

This has the general solution

$$\phi(x) = C_1e^{\beta_1x} + C_2e^{\beta_2x}, \quad (21)$$

where β_1 and β_2 are the roots of the fundamental quadratic,

$$\beta = \frac{\mu}{\sigma^2} - \frac{1}{2} \pm \frac{\sqrt{2\lambda\sigma^2 + (\mu - \sigma^2/2)^2}}{\sigma^2} \quad (22)$$

and where $\beta_1 > 0 > \beta_2$. The coefficients C_1 and C_2 are solved below.

Case 2: $x \in (x_a, x_b)$

In this region, firms exit at rate λ due to the death shock and a flow of new firms enter with initial log cash flows given by the distribution $g(x)$. Over this region, the stationary distribution $\phi(x)$ satisfies the KFE:

$$\frac{1}{2}\sigma^2\phi_{xx}(x) - \left(\mu - \frac{1}{2}\sigma^2\right)\phi_x(x) - \lambda\phi(x) + g(x) = 0. \quad (23)$$

This has the general solution

$$\phi(x) = D_1e^{\beta_1x} + D_2e^{\beta_2x} + D_3e^x, \quad (24)$$

where β_1 and β_2 are the same roots of the fundamental quadratic given in Equation (22) of Case 1 above. We can solve for D_3 , the coefficient on the particular solution of the KFE, by plugging in:

$$\frac{1}{2}\sigma^2D_3e^x - (\mu - \frac{1}{2}\sigma^2)D_3e^x - \lambda D_3e^x + e^{x-\hat{x}} = 0. \quad (25)$$

This can be rearranged as

$$D_3 = \frac{e^{-\hat{x}}}{\lambda + (\mu - \frac{1}{2}\sigma^2) - \frac{1}{2}\sigma^2}, \quad (26)$$

which gives a general solution in this case of

$$\phi(x) = D_1 e^{\beta_1 x} + D_2 e^{\beta_2 x} + \frac{e^{x-\hat{x}}}{\lambda + (\mu - \frac{1}{2}\sigma^2) - \frac{1}{2}\sigma^2}. \quad (27)$$

The coefficients D_1 and D_2 are solved by imposing the boundary conditions given below.

Case 3: $x \in (x_b, \bar{x})$

As in case 1, firms in this region exit at rate λ and there is no new entry in this region, so we have the same ODE characterizing the KFE. That is, the KFE satisfies

$$\frac{1}{2}\sigma^2 \phi_{xx}(x) - \left(\mu - \frac{1}{2}\sigma^2\right) \phi_x(x) - \lambda \phi(x) = 0, \quad (28)$$

and the general solution is given by

$$\phi(x) = H_1 e^{\beta_1 x} + H_2 e^{\beta_2 x}, \quad (29)$$

where again the coefficients H_1 and H_2 are solved for by imposing the appropriate boundary conditions.

We have a total of six boundary conditions for the stationary distribution of log cash flows of private firms, $\phi(x)$:

$$\phi(x_D) = 0 \quad (30)$$

$$\phi(\bar{x}) = 0 \quad (31)$$

$$\lim_{x \uparrow x_a} \phi(x) = \lim_{x \downarrow x_a} \phi(x) \quad (32)$$

$$\lim_{x \uparrow x_a} \phi_x(x) = \lim_{x \downarrow x_a} \phi_x(x) \quad (33)$$

$$\lim_{x \uparrow x_b} \phi(x) = \lim_{x \downarrow x_b} \phi(x) \quad (34)$$

$$\lim_{x \uparrow x_b} \phi_x(x) = \lim_{x \downarrow x_b} \phi_x(x). \quad (35)$$

Equations (30) and (31) follow from the fact that private firms exit when their log cash flow falls to x_D and choose to go public when their log cash flows reach the IPO threshold \bar{x} . Equations (32)–(35) ensure sufficient smoothness for $\phi(x)$. These six boundary conditions determine the six coefficients, $C_1, C_2, D_1, D_2, H_1, H_2$.

The stationary distribution of private firm log cash flows, $\phi(x)$, is given by

$$\phi(x) = \begin{cases} C_1 e^{\beta_1 x} + C_2 e^{\beta_2 x}, & \text{if } x_D < x < x_a \\ D_1 e^{\beta_1 x} + D_2 e^{\beta_2 x} + \frac{e^{x-\hat{x}}}{\lambda + (\mu - \frac{1}{2}\sigma^2) - \frac{1}{2}\sigma^2}, & \text{if } x_a \leq x \leq x_b \\ H_1 e^{\beta_1 x} + H_2 e^{\beta_2 x}, & \text{if } x_b < x < \bar{x}. \end{cases} \quad (36)$$

For the *level* of cash flows, X , the stationary distribution of private firms, $\varphi(X)$, can be expressed as

$$\varphi(X) = \begin{cases} C_1 X^{\beta_1-1} + C_2 X^{\beta_2-1}, & \text{if } X_D < X < X_a \\ D_1 X^{\beta_1-1} + D_2 X^{\beta_2-1} + \frac{1}{(X_b - X_a)(\lambda + (\mu - \frac{1}{2}\sigma^2) - \frac{1}{2}\sigma^2)}, & \text{if } X_a \leq X \leq X_b \\ H_1 X^{\beta_1-1} + H_2 X^{\beta_2-1}, & \text{if } X_b < X < \bar{X}, \end{cases} \quad (37)$$

3.4 Quantity of private firms, IPO rate

Letting N denote the exogenous flow of new private firms entering, the aggregate mass of private firms in steady state, $Q_{private}$, is given by

$$Q_{private} = N \int_{x_D}^{\bar{x}} \phi(x) dx, \quad (38)$$

where $\phi(x)$ is given in Equation (36). Let Υ denote the steady state flow of IPOs. Given the distribution of private firms, the flow rate of IPOs can be computed as

$$\Upsilon = -\frac{1}{2}\sigma^2 N \phi'(\bar{x}) = -\frac{1}{2}\sigma^2 N (\beta_1 H_1 e^{\beta_1 \bar{x}} + \beta_2 H_2 e^{\beta_2 \bar{x}}). \quad (39)$$

3.5 Distribution of Public firms

In steady state, there is a flow Υ of firms becoming public, each entering the public firm distribution with an initial revenue of \bar{X} , determined by the optimal IPO threshold. Upon becoming public, the firm's cash flows then evolve according to the previously specified cash flow dynamics for a public firm. Public firms exit when their cash flows drop to $X_{D,pub}$ or when hit with a death shock, which is a Poisson arrival with intensity λ_2 . Thus, the distribution of public firms has support $(X_{D,pub}, \infty)$.

Let $\psi(x)$ denote the stationary distribution of log cash flows of public firms. This distribution satisfies the KFE

$$\frac{1}{2}\sigma_2^2\psi_{xx}(x) - \left(\mu_2 - \frac{1}{2}\sigma_2^2\right)\psi_x(x) - \lambda_2\psi(x) = 0, \quad (40)$$

for $x \neq \bar{x}$. The general solution is given by

$$\psi(x) = \begin{cases} J_1e^{\zeta_1x} + J_2e^{\zeta_2x}, & \text{if } x_{D,public} < x < \bar{x} \\ K_1e^{\zeta_1x} + K_2e^{\zeta_2x}, & \text{if } x > \bar{x}, \end{cases} \quad (41)$$

where ζ_1 and ζ_2 are the roots of the fundamental quadratic,

$$\zeta = \frac{\mu_2}{\sigma_2^2} - \frac{1}{2} \pm \frac{\sqrt{2\lambda_2\sigma_2^2 + (\mu_2 - \sigma_2^2/2)^2}}{\sigma_2^2}, \quad (42)$$

with $\zeta_1 > 0 > \zeta_2$. The coefficients J_1 , J_2 , K_1 , and K_2 are determined by imposing the following conditions:

$$\int_{\bar{x}}^{\infty} \psi(x) < \infty \quad (43)$$

$$J_1e^{\zeta_1x_{D,pub}} + J_2e^{\zeta_2x_{D,pub}} = 0 \quad (44)$$

$$J_1e^{\zeta_1\bar{x}} + J_2e^{\zeta_2\bar{x}} = K_2e^{\zeta_2\bar{x}} \quad (45)$$

$$\lambda_2\Upsilon \left(\int_{x_{D,pub}}^{\infty} \psi(x)dx \right) - \frac{1}{2}\sigma_2^2\Upsilon\psi'(x_{D,pub}) = \Upsilon \quad (46)$$

Equation (43) ensures that $\psi(x)$ is integrable as $x \rightarrow \infty$, which implies $K_1 = 0$. Equation (44) states that there is zero mass of public firms at the optimal exit boundary, $x_{D,pub}$ and equation (45) ensures continuity of $\psi(x)$ at the IPO entry point, \bar{x} .

Equation (46) follows from the definition of a steady state distribution of public firms. In steady state, the flow of exit is equal to the flow of entry. The mass of public firms, Q_{public} , can be expressed as

$$Q_{public} = \Upsilon \left(\int_{x_{D,pub}}^{\infty} \psi(x)dx \right). \quad (47)$$

Of the existing mass of public firms, a fraction λ_2 exit due to the Poisson shock. Additionally, some public firms exit by hitting the lower bound of cash flows, $x_{D,public}$, at which they

optimally abandon. The flow of firms hitting this lower threshold $x_{D,public}$ is given by

$$-\frac{1}{2}\sigma_2^2\Upsilon\psi'(x_{D,pub}). \quad (48)$$

Together, these two forms of exit account for the left hand side of Equation (46). This flow of exit must be equal to the inflow of public firms, which is the flow of private firms exercising their IPO option, Υ .

So the stationary distribution of public firm log cash flows is given by $\Upsilon \times \psi(x)$, where

$$\psi(x) = \begin{cases} J_1e^{\zeta_1x} + J_2e^{\zeta_2x}, & \text{if } x_{Dpub} < x < \bar{x} \\ K_2e^{\zeta_2x}, & \text{if } x > \bar{x} \end{cases} \quad (49)$$

The distribution of the level of cash flows, X for public firms is given by $\Upsilon \times \Psi(X)$, where

$$\Psi(X) = \begin{cases} J_1X^{\zeta_1-1} + J_2X^{\zeta_2-1}, & \text{if } X_{Dpub} < X < \bar{X} \\ K_2X^{\zeta_2-1}, & \text{if } X > \bar{X} \end{cases} \quad (50)$$

Figure 11 displays the distribution of private (Panel A) and public (Panel B) firms from the model. These distributions are under the parameters of the early period calibration, listed in the first column of Table 2 and described in the next section.

4 Calibration and Model Results

In this section we calibrate the parameters of the model to match data moments for the early period of our sample, 1980–1996. We then use the calibrated model to evaluate how changes in underlying parameters affect moments of interest.

There are a total of eleven parameters to calibrate. Both private and public firms each have five parameters: the growth rate (μ) and volatility (σ) of cash flows, the discount rate (r), the intensity of the Poisson exit shock (λ), and the fixed cost of operation (c). The final parameter is the fixed cost of the IPO, \mathcal{I}_{IPO} . As a starting point, we assume that private and public cash flow growth and volatility are equal: $\mu_1 = \mu_2$ and $\sigma_1 = \sigma_2$. We normalize the value of the private firm's fixed cost to a value of $c_1 = 5$. The difference in the public

firm fixed cost and the private firm fixed cost, $c_2 - c_1$, reflects the relative increase in the ongoing costs of being a publicly listed firm. This leaves eight parameters to calibrate.

Table 1 shows the moments used to calibrate the model for the early period of 1980–1996. We use a total of 11 moments to calibrate the remaining eight parameters of the model. In general, the model moments depend on multiple parameters so that no single moment uniquely identifies a particular parameter. The limited availability of private firm data requires us to rely on public firm data as well as features of the IPO data to calibrate the private firm parameters. For example, the average age and size of firms at IPO, as well as the rate of IPOs relative to the mass of public firms provides information on the private firm parameters. Comparing the model and data columns, we see that the model does a reasonably good job of matching these moments that were targeted in the calibration.

Table 2 displays the calibrated model parameters. Comparing the two columns, we see that many of the model parameters differ significantly between the two periods. Private firms face a lower fixed cost of operation but a higher discount rate compared to public firms. Additionally, private firms face a higher intensity of the Poisson exit shock, consistent with the fact, shown in Table 1, that private firms have a higher turnover rate than public firms.

We now use the calibrated model to evaluate which parameter changes correspond to the changes observed in the data over the last two decades. We focus our attention on how a change in the fixed cost of being public, c_2 , the sunk cost to becoming public, \mathcal{I}_{IPO} , and private firms cost of capital, r_1 , affect moments of interest. We consider two values of change for each of these parameters. For each case, we compute a set of model moments and report the ratio of these moments to their counterpart values in the benchmark calibration. In each case, we change only the one model parameter, holding all other parameter values fixed at their benchmark value reported in Table 2. In Table 4 we display the results of these parameter changes. Each column corresponds to a different changed parameter value and the moment values reported are relative to the moments in the benchmark calibration. That is, a value of one in the Table means that the model moment is the same for the changed

parameter case as in the benchmark calibration.

The first two columns of Table 4 show the effect of a reduction in private firms' cost of capital, r_1 . For a lower private firm discount rate, the relative value to becoming a public firm is lower. As a result, there are fewer IPOs and fewer public firms. With a lower value of r_1 , private firms optimally wait longer to exercise their IPO option. The result is that firms are larger at the time of their IPO. While there are fewer public firms, the average public firm size is larger when there is a lower private cost of capital.

The third and fourth columns of Table 4 show the effect of an increase in the fixed operating costs for public firms, c_2 . This change also makes the prospect of becoming a publicly listed firm less attractive. As a result, the qualitative effects are largely similar to those in the case of a reduced private firm cost of capital, however, the magnitudes of the effects are different in several cases. The final two columns report the effect of a change in the sunk cost of becoming a public firm, \mathcal{I}_{IPO} . Again, many of the effects are qualitatively similar but differ in magnitude for the moments reported in the table.

5 Conclusion

We investigate the puzzling drop in the number of public firms over 1996 to present. There are a variety of empirically supported mechanisms, yet no consensus on which are most important. We attempt to fill this gap. First, we hone in on a set of empirical facts helpful in shaping a story for the decline. Second, we develop a model that clarifies the forces underlying the going public decision. Last, we do a calibration and compute counterfactuals in order to distinguish among qualitatively similar mechanisms and quantify effects of parameters.

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Tables

	Model	Data: 1980–1996
Average Age at IPO	17.068	15.000
IPO Value / Mean public firm value	0.242	0.190
IPO Value / Median public firm value	0.823	0.780
IPOs / # Public firms	0.060	0.048
Fraction of Public firms < IPO Value	0.416	0.470
Total Public Value / Total Public Earnings	10.039	7.650
Public firm Pareto right tail slope coefficient (ζ_2)	-1.304	-1.250
Average Age of Public Firms	22.222	19.180
Turnover rate for all firms	0.184	0.193
Turnover rate for public firms	0.121	0.113
Median public firm sales growth	0.025	0.019

Table 1: **Moments used to calibrate the model.** The table reports the moments used to calibrate the parameters of the model. The first column shows the moments computed from the model and the second column reports the target moments from the data for the period 1980–1996. Reported values are time series averages over their respective period. Where applicable, values are reported in annualized units.

<i>Private firms</i>	
μ_1	0.025
σ_1	0.25
r_1	0.11
λ_1	0.10
c_1	5
<i>Public firms</i>	
μ_2	0.025
σ_2	0.25
r_2	0.085
λ_2	0.045
c_2	6.5
\mathcal{I}_{IPO}	7

Table 2: **Model parameter values.** The table reports the calibrated model parameter values for the period 1980–1996. Parameters are in annualized units where applicable and the moments used for calibration are listed in Table 1.

	Early Period	Late Period	Late / Early
IPOs/Number public firms	0.048	0.021	0.44
IPOs/Number private firms	0.043	0.008	0.18
Number of IPOs	293	100	0.34
Number of public firms	5799	4752	0.82
Aggregate valuation of public firms (\$ trillions)	12.0	18.3	1.52
Public firms / All (> 500) firms	0.44	0.27	0.61
Median public firm value	249	574	2.30
Average Public firm value (\$ millions)	2895	6227	2.15
Value at IPO (median)	194	660	3.4

Table 3: **Comparison of data moments for early and late period.** The table compares moments from the data for the early (1980–1996) and late (2001–2013) periods. The final column reports the ratio of the moment in the late period relative to the early period. Where applicable, the moments are reported in annualized units. See Section 2 for a description of the data.

	$r_1 = 0.1$	$r_1 = 0.09$	$c_2 = 7.5$	$c_2 = 8.5$	$\mathcal{I}_{IPO} = 8.5$	$\mathcal{I}_{IPO} = 10.5$
IPOs	0.890	0.771	0.660	0.469	0.902	0.795
IPOs / public mass	0.978	0.954	0.974	0.957	0.981	0.959
IPOs / private mass	0.813	0.644	0.521	0.330	0.839	0.686
Value at IPO	1.108	1.251	1.413	1.829	1.094	1.216
IPO Cost / Value at IPO	0.902	0.799	0.708	0.547	1.110	1.233
Age at IPO	1.009	1.001	0.975	0.879	1.009	1.004
Mass of public firms	0.910	0.808	0.677	0.490	0.920	0.828
Mass of private firms	1.094	1.197	1.266	1.421	1.075	1.158
Total public value	0.963	0.918	0.837	0.722	0.967	0.926
Total private value	1.181	1.412	1.419	1.732	1.109	1.239
Average public value	1.059	1.136	1.236	1.473	1.051	1.117
Average private	1.080	1.179	1.121	1.219	1.031	1.069
Median public value	1.082	1.190	1.358	1.719	1.071	1.163
Median private value	1.142	1.333	1.360	1.728	1.082	1.190
% public earnings < 0	0.914	0.820	0.897	0.831	0.924	0.841
% public < IPO value	1.021	1.044	1.025	1.041	1.018	1.039

Table 4: **Changes in moments for underlying parameters.** The table reports the ratio of model moments under alternative parameters relative to their benchmark values. Each column displays the case for a single parameter value that is changed, with all other parameters held at their benchmark values reported in Table 2. The values reported are the ratio of the model value in this alternative parameterization to the value in the benchmark. A value equal to one indicates that the moment is unchanged by the parameter change.

Figures

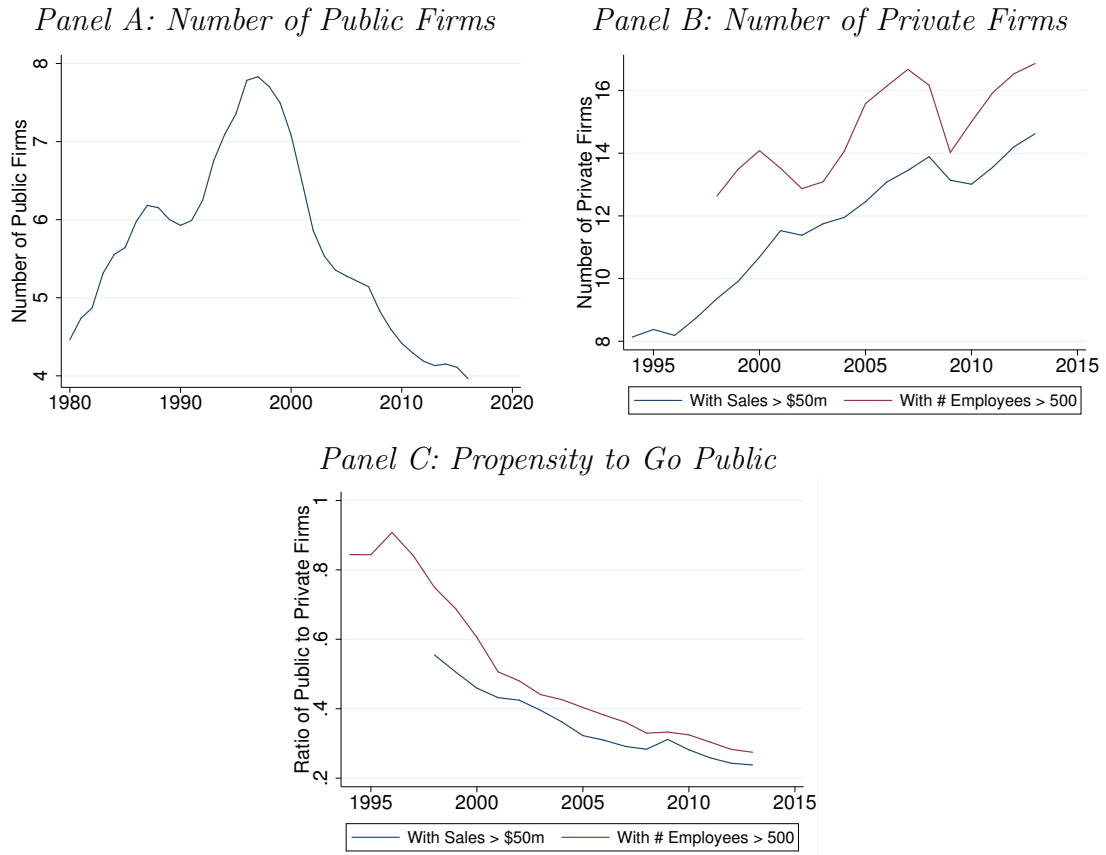


Fig. 1. Publicly Listed and Private firms.

Panel A shows the count of publicly listed U.S. firms. Panel B displays the number of U.S. private firms with sales above \$50m and with more than 500 employees. Panel D shows the ratio of the number of publicly listed firms to the number of private firms with sales above \$50m. Panel C shows the ratio of the number of publicly listed firms to the number of private firms. The firm counts are expressed in thousands.

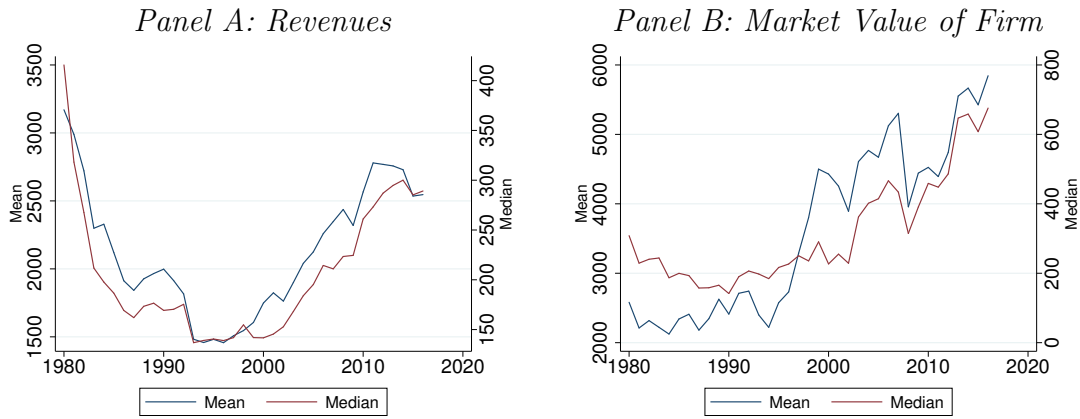


Fig. 2. Average and Median Size of Public Firms

The Figure plots the mean and median size of publicly listed U.S. firms. In Panel A, the firm's size is measured with revenues. In Panel B, the firm's size is measured with market value of firm. The data are real annual observations from 1980 to 2016, and are expressed in millions of December 2005 dollars.

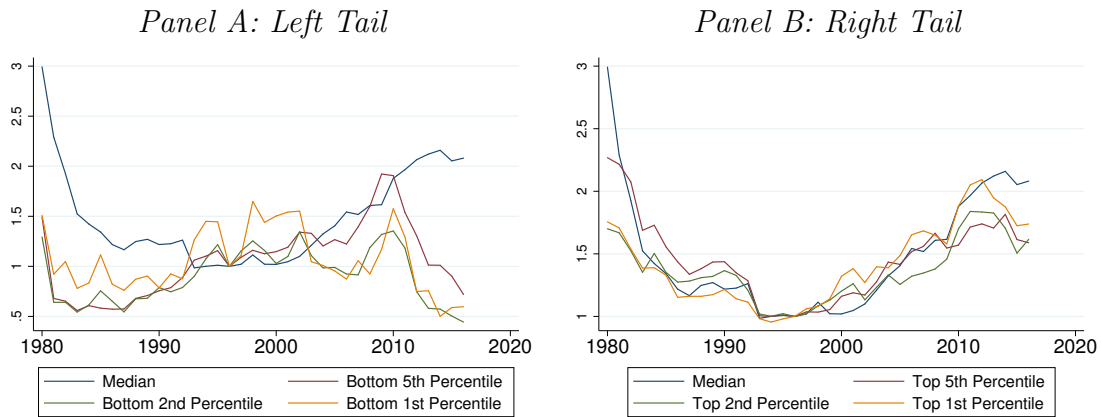


Fig. 3. Tails of Public Firms' Distribution

The Figure shows the revenues in the bottom and top 1st, 2nd, 5th percentiles versus the median revenues of publicly listed U.S. firms (Panels A and B, respectively). The data are real annual observations from 1980 to 2016, and indexed to 1996.

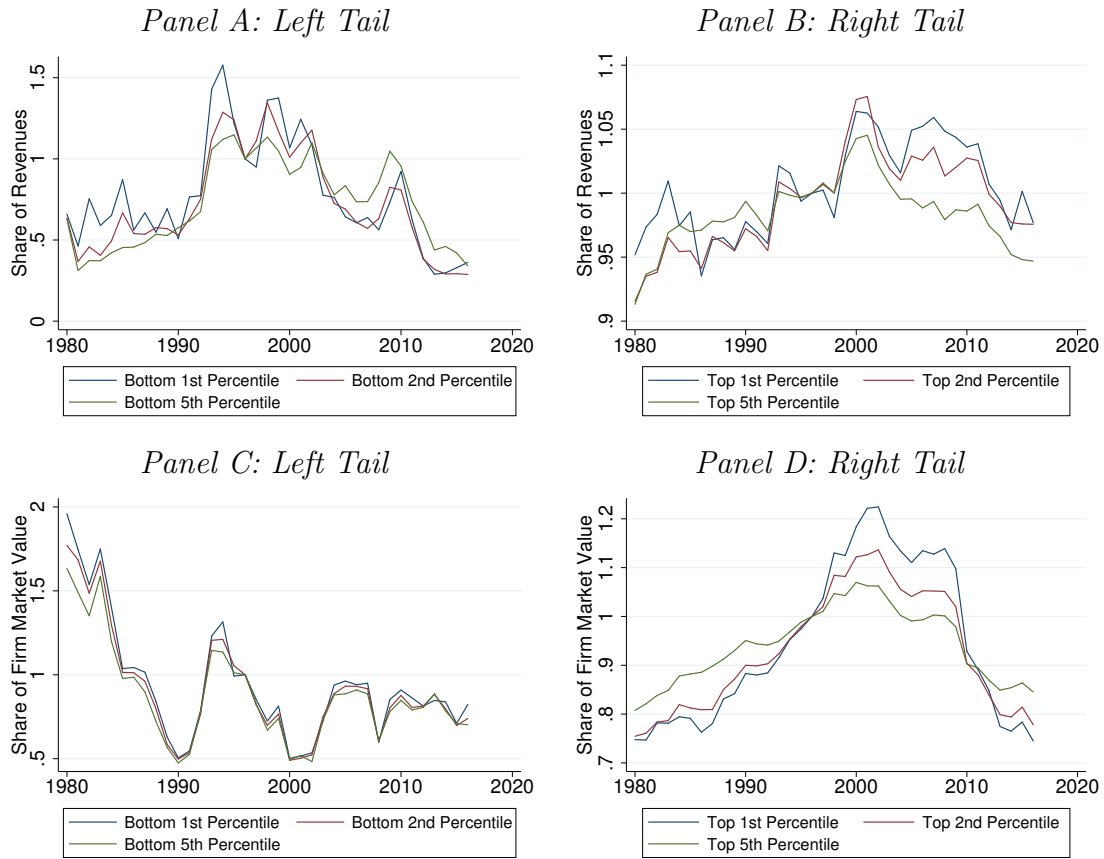


Fig. 4. Tails of Public Firms' Distribution

The Figure shows the share of revenues and firm market value from publicly listed U.S. firms in the bottom and top 1st, 2nd, 5th percentiles. The data are real annual observations from 1980 to 2016, and indexed to 1996.

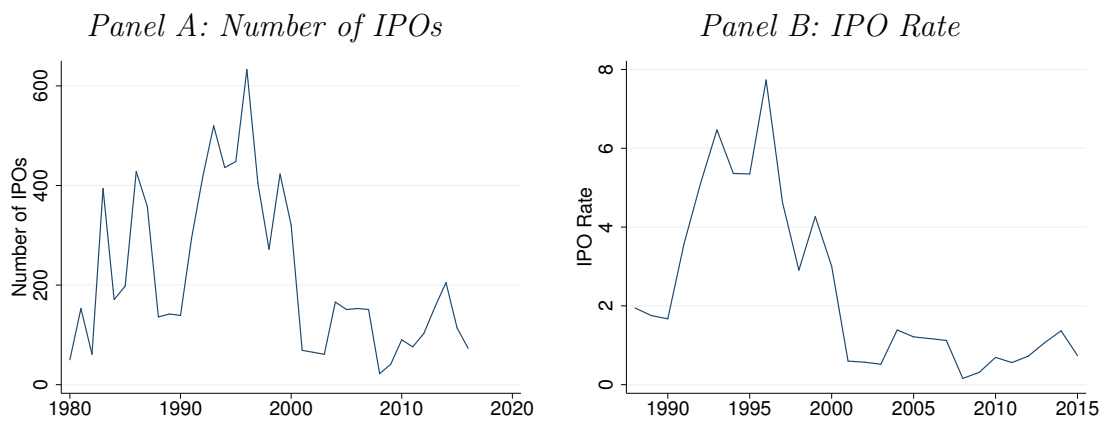


Fig. 5. Entry Rate of Public Firms

The Panel A shows the number of IPOs. The data are annual observations from 1980 to 2016. Panel B shows the IPO rate, which is calculated as the ratio of the number of IPOs to the number of private firms with more than 500 employees. The data are annual observations from 1988 to 2015, and expressed in percentages.



Fig. 6. Exit Rate of Public Firms

Panel A shows the number of firm exits. Panel B shows the exit rate, which is calculated as the ratio of the number of delists to the total number of public firms. Panel C shows the number of exits for “negative” reasons. Panel D shows the exit rate for “negative” delist. Panel E shows the number of mergers. Panel D shows the exit rate via mergers and acquisitions. The data are annual observations from 1980 to 2016. In Panels B, D, and F, the data are expressed in percentages. The “negative” delists are defined as securities with delisting code *4xx* and *5xx*. Mergers and acquisitions are defined as securities with delisting code *2xx* and *3xx*.

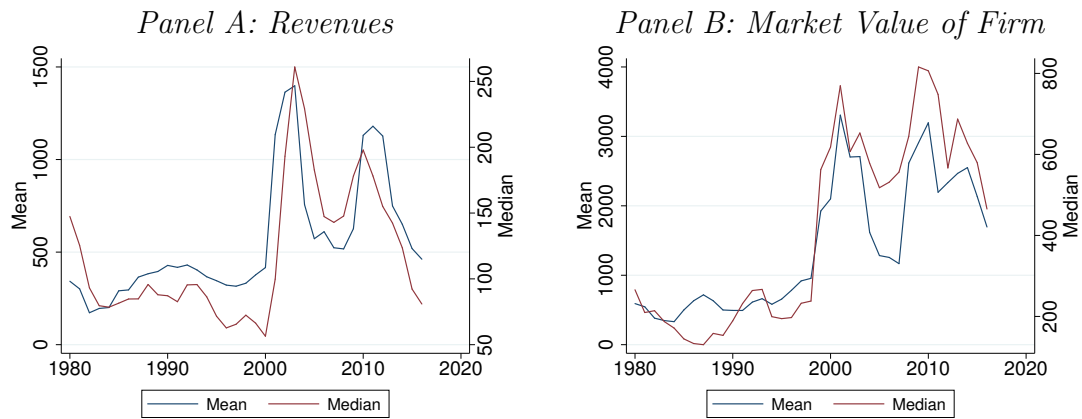


Fig. 7. Average and Median Size of Firms at IPO

The Figure plots the mean and median size of firms at IPO. The means and medians are smoothed using the three-year moving average. In Panel A, the firm's size is measured with revenues. In Panel B, the firm's size is measured with market value of firm. The data are real annual observations from 1980 to 2016, and are expressed in millions of December 2005 dollars.

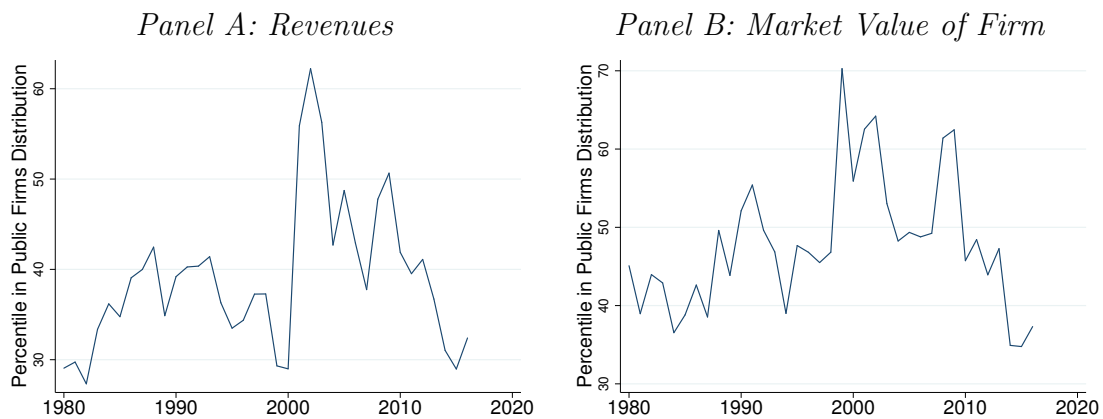


Fig. 8. Share of Public Firms Below Median Firm Size at IPO

The Figure plots the share of public firms below the median firm's size at IPO. In Panel A, the firm's size is measured with revenues. In Panel B, the firm's size is measured with market value of firm. The data are annual observations from 1980 to 2016, and are expressed in percentages.

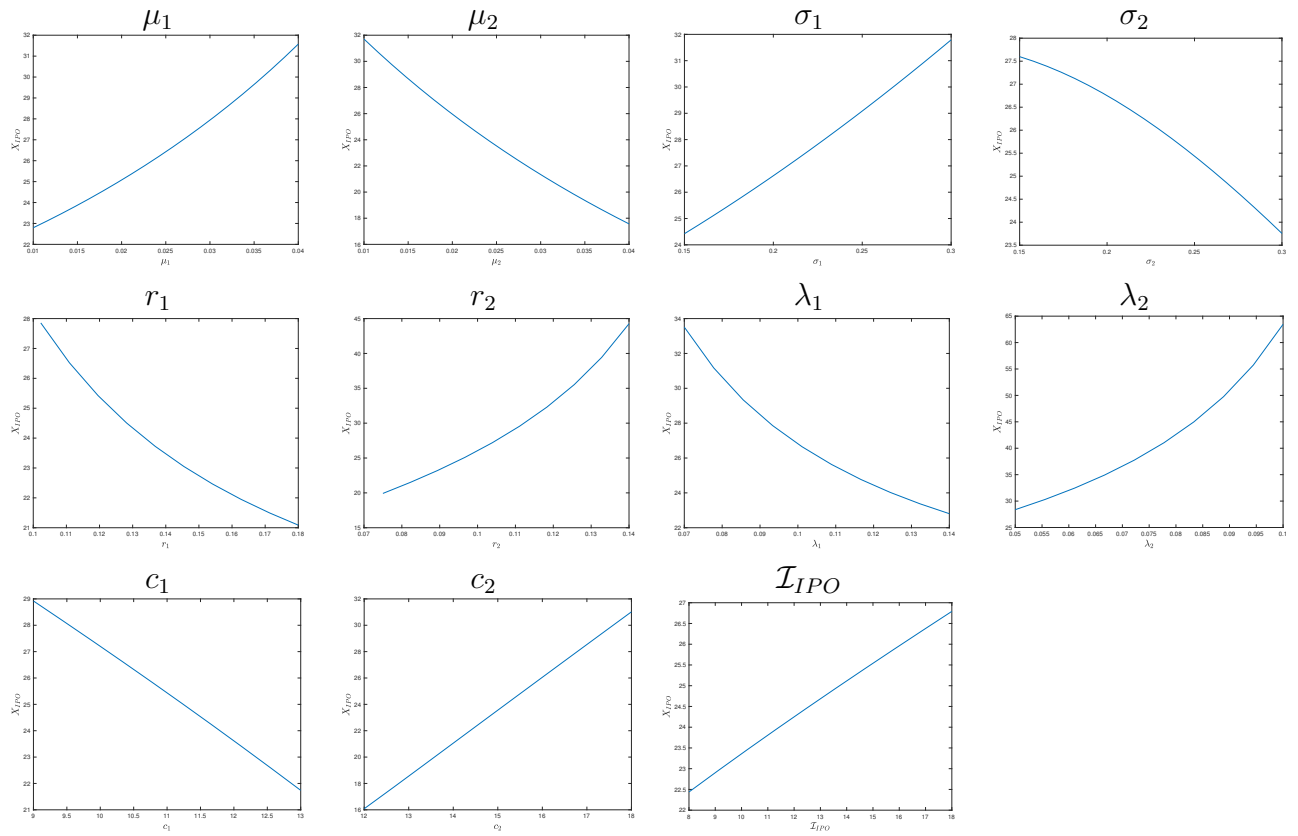


Fig. 9. Comparative statics for IPO threshold.

The figure displays a firm's choice of optimal IPO threshold, X_{IPO} , as a function of underlying parameters.

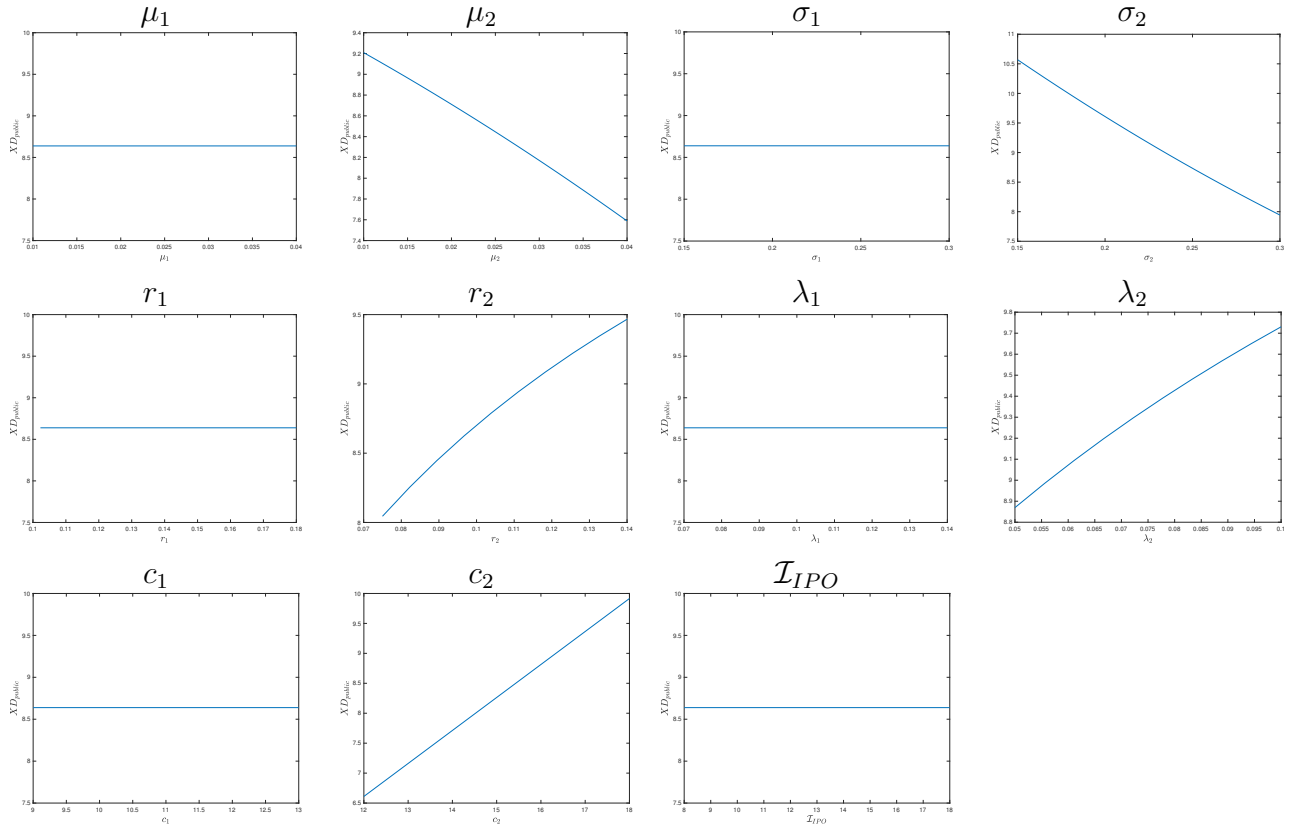
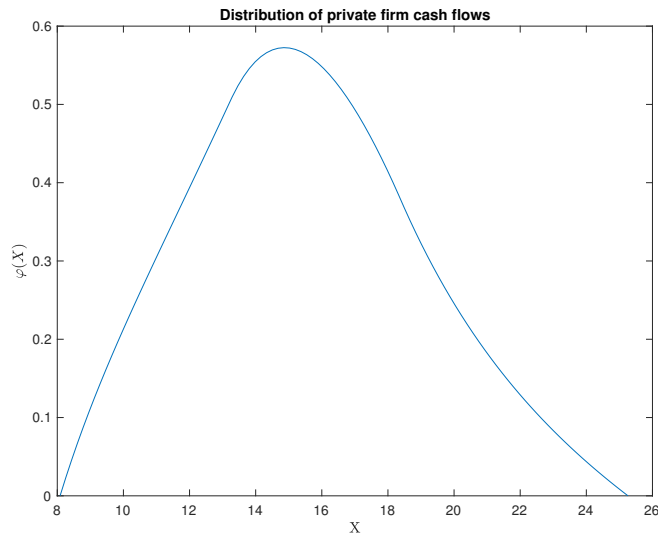


Fig. 10. Comparative statics for public firm exit threshold.

The figure displays the optimal exit threshold for public firms, $X_{D,public}$, as a function of underlying parameters.

Panel A: Private firm distribution



Panel B: Public firm distribution

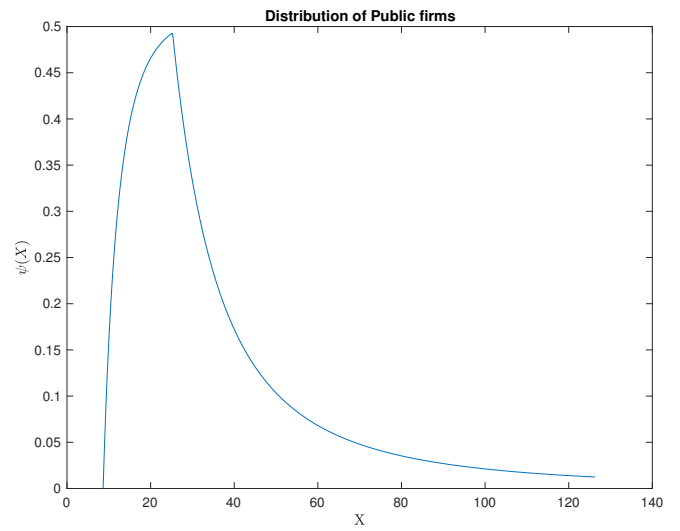


Fig. 11. Distribution of Private and Public firms

The figure displays the distribution of private firms (Panel A) and public firms (Panel B) from the model under the parameters calibrated from the early sample period, 1980–1996.

6 Data Appendix

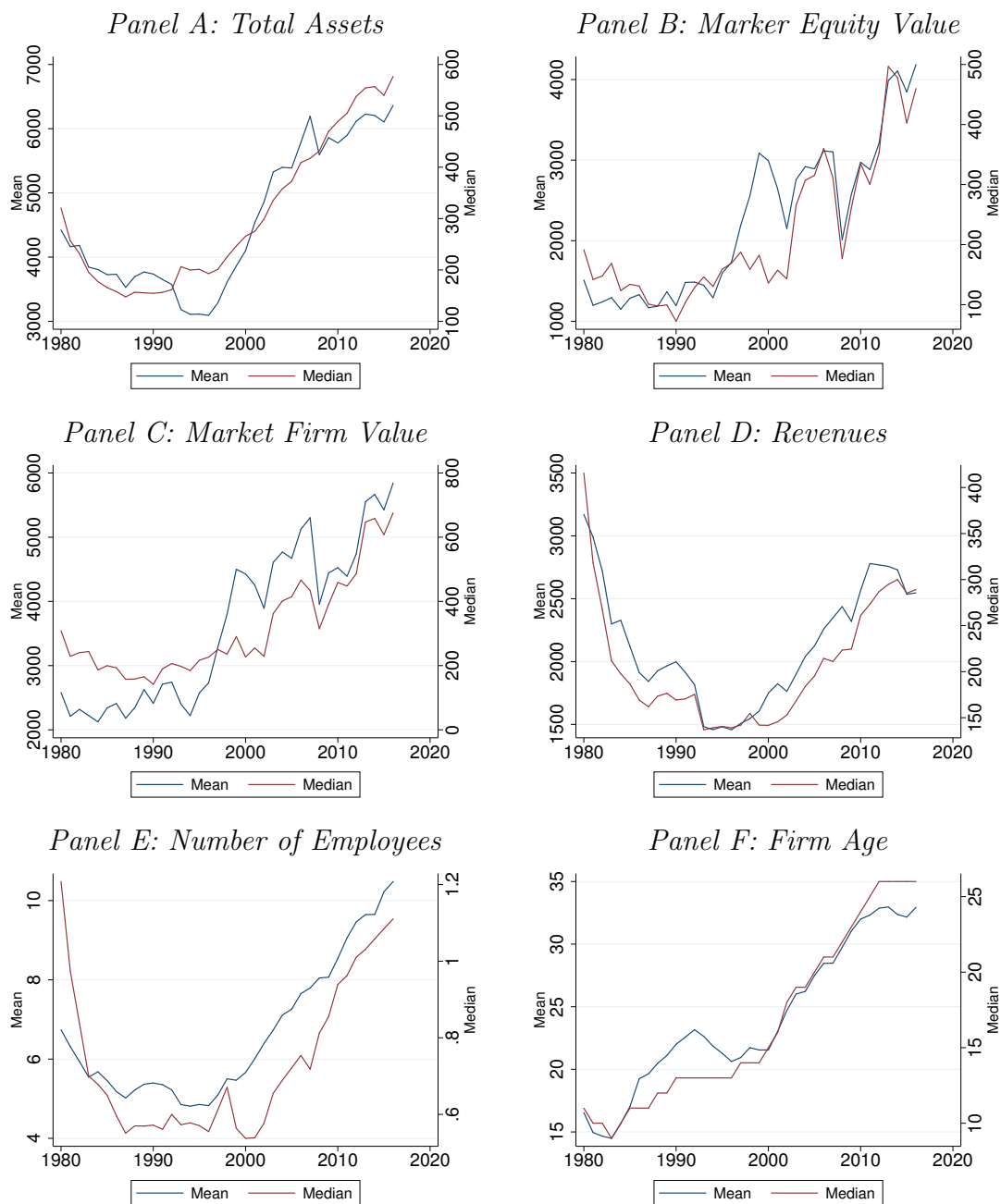


Fig. A.1. Average and Median Size of Public Firms

The Figure shows the mean and median size of publicly listed U.S. firms. The firm's size is measured with total assets, market value of equity, market value of firm (the sum of market value of equity and book value of debt), revenues, number of employees, and age. In Panels A - D, the data are real annual observations from 1980 to 2016, and are expressed in millions of December 2005 dollars. In Panel E, the data are annual observations, expressed in thousands. In Panel F, the data are annual observations, expressed in years.

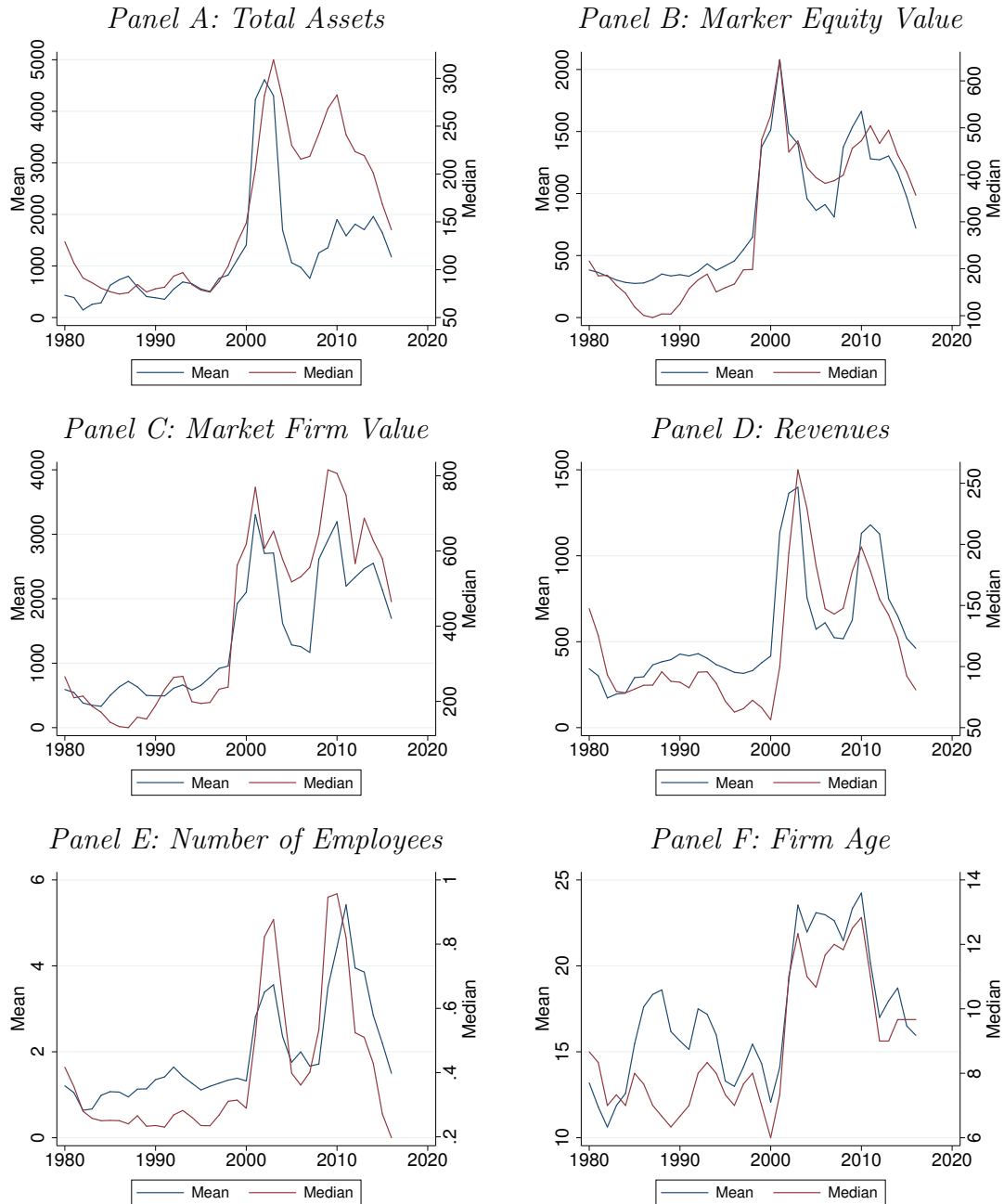


Fig. A.2. Average and Median Size of Firms at IPO

The Figure shows the mean and median size of firms at IPO. The means and medians are smoothed using the three-year moving average. The firm's size is measured with total assets, market value of equity, market value of firm (the sum of market value of equity and book value of debt), revenues, number of employees, and age. In Panels A - D, the data are real annual observations from 1980 to 2016, and are expressed in millions of December 2005 dollars. In Panel E, the data are annual observations, and are expressed in thousands. In Panel F, the data are annual observations, and are expressed in years.